

For this experiment the end wall was removed from the tail-piece (fig. 3), and a large flexible hoop substituted. By this means, it was hoped that when the whole was placed in the bath it would be possible, by mere expansion of the hoop, to obtain a clean surface in the well. The event proved, however, that the purification did not proceed readily beyond the earlier stages, unless the passage of the contamination through the long channel of the tail-piece was facilitated by wind.]

V. "Experiments with Lord Rayleigh's Colour Box." By
ARTHUR SCHUSTER, F.R.S. Received May 15, 1890.

Lord Rayleigh described before the meeting of the British Association, in 1881,* a colour box in which artificial yellow is produced by mixing a pure red and green, and this yellow is directly compared to the yellow of the spectrum. Lord Rayleigh has given an account of certain peculiarities of vision observed in a number of persons, and it seemed to me worth while to extend the enquiry to a greater number of observers, and also, if possible, to obtain some evidence as to the existence of smaller differences than those described in Lord Rayleigh's paper.

The instrument used was made according to Lord Rayleigh's second pattern, in which a double-image prism is interposed between the slit and collimator lens; the prism which separates the light being a direct-vision prism. For the detailed description of the instrument I must refer the reader to Lord Rayleigh's paper.

My attention was in the first instance directed to prove or disprove the existence of small differences in different persons. It was necessary, therefore, only to take persons in whose observing powers I could place some reliance, and, secondly, to multiply the number of observations of each individual, so as to obtain an idea of the degree of accuracy to which the observations could be trusted. The instrument was used in a fairly dark room, and the observer was asked to place the Nicol so as to obtain the required match. After the reading had been taken the Nicol was displaced and the observations repeated. Five separate readings were thus generally obtained, and occasionally more. Often separate sets of observations were made for the right and left eye. As, owing to imperfections of construction, the zero of the instrument did not remain constant, either myself or Mr. Hadley, one of the demonstrators in the Physical Laboratory, took a reading whenever observations were made.

I have often compared my vision with Mr. Hadley's, and never detected any difference amounting to more than 0.1 of a division of the

* 'Nature,' Nov. 17, 18 1.

scale of my instrument; so that I have taken our colour matches as equal, and referred all others to them, although, as will appear, we do not quite agree with the majority of people. In order to understand the numbers given in the following pages, it is necessary to state that in the neighbourhood of normal vision a difference of one-tenth of a division means a difference of about $2\frac{1}{2}$ per cent. in the ratio of intensities of red to green. I consider that the mean of five readings of a practised observer should not differ by more than that on different occasions. If two good observers place the Nicol half a division different from each other, then each should be able in his own mind to be certain that the other's match is wrong. The difference of a whole division is generally very obvious after a little practice with the instrument.

It appears that differences of between a half and whole division are very common, so that there cannot be any doubt of the real existence of small differences, possibly following, as far as the number of observations allow us to say, the ordinary law for deviations from a mean. But the larger differences, such as Lord Rayleigh was the first to observe, seem certainly to be more frequent than the distribution of small differences would lead us to expect. As has already been stated, it seemed better in the first instance to confine myself to a careful examination of a limited number of cases than to extend the enquiry too much before I could form an estimate of the accuracy which is to be expected from a casual observer. I have examined seventy-five: of these three proved colour blind; four, of which three belonged to the same family, showed the same peculiarity of vision as Mr. Balfour and Professor J. J. Thomson, while one showed a large difference in the opposite direction.

In the following tables I shall call my own reading zero, and shall take as the *unit* difference between myself and others *one-tenth of a division* of the divided circle. The first table gives the ratio of red to green used in the match, corresponding to the stated differences of reading:—

Table I.

—70	6.15	+ 2	0.82
—60	4.37	+ 4	0.78
—50	3.21	+ 6	0.74
—40	2.41	+ 8	0.70
—30	1.84	+10	0.67
—20	1.42	+20	0.51
—10	1.11	+30	0.39
—8	1.05	+40	0.29
—6	1.00	+50	0.21
—4	0.95	+60	0.15
—2	0.90	+70	0.10
±0	0.86		

It will be seen from this table that the proportion of red to green which I require to make an artificial yellow is 0·86. This number is not comparable with those given by Lord Rayleigh for his own and Mr. Balfour's vision, as everything depends on the particular green which he uses to make the match. I was aiming in setting up the instrument to choose such a red and green as would without the Nicol make a yellow, and perhaps took too much of a yellowish-green in consequence.* Table II exhibits the results obtained for various observers. The means only of all observations taken at different times, and with both eyes, are given.

Table II.

Observer.	Number of readings.	Mean of readings.	Mean difference of readings.
1	6	0	1·0
2	17	-1·0	2·3
3	10	+1·5	5·3
4	25	0	1·9
6	12	-4·0	4·4
7	5	+3·0	3·4
8	5	-1·0	2·8
9	20	-4·0	1·8
10	5	-7·0	2·2
11	5	+2·0	3·6
12	7	+3·3	2·0
13	5	0	1·0
14	15	-7·3	2·3
15	15	+3·0	2·7
16	6	-11·5	0·9
18	5	-5·0	3·6
19	5	-5·0	4·0
20	5	-12·0	2·0
21	20	-7·2	3·1
22	5	-4·0	2·8
23	12	-1·5	4·3
24	5	-11·0	1·6
25	10	+3·5	2·9
26	20	-5·3	1·8
27	20	0	1·3
29	18	+1·2	2·0

* [Note, added May 23.—Since writing the above, I have determined the wave-lengths of the colours selected, and was surprised to find how much more yellow the green was than I thought or intended; the wave-length of the green which was mixed with the red was about $5\cdot620 \times 10^{-5}$. The red was a full red near C, the yellow about the sodium yellow.]

Table II—*continued.*

Observer.	Number of readings.	Mean of readings.	Mean difference of readings.
30	23	-5.0	1.4
31	14	-9.0	2.6
32	10	-2.5	1.8
33	8	-2.0	1.6
34	9	-1.5	2.3
35	10	-3.0	2.4
36	10	-15.0	1.2
37	20	-5.5	2.1
38	10	-3.5	1.8
39	10	0	2.6
40	10	-10.0	2.1
41	20	9.5	2.1
42	26	-6.7	2.4
43	10	-2.0	1.1
44	10	-14.0	2.2
45	20	-2.2	2.2
47	10	-11.0	3.0
48	10	-5.5	2.7
49	10	-2.0	1.8
51	10	-1.5	2.8
52	10	-6.5	5.0
53	10	-2.5	5.0
54	10	-3.0	1.5
55	10	-8.5	3.5
56	10	-4.5	2.8
57	10	-1.0	7.5
58	10	-16.0	3.4
59	10	-13.5	2.5
60	10	-14.0	3.2
61	5	-11.0	0.8
62	5	+3.0	2.0
63	8	-1.0	1.7
64	5	-6.0	1.2
65	5	-4.0	2.8
67	15	-4.0	2.8
68	5	-9.0	2.8
69	5	-2.0	3.0
71	10	-4.0	2.9
73	10	+1.0	1.1
74	10	-3.0	3.0
75	6	-5.0	

Table II—*continued*.

Observer.	Number of readings.	Mean of readings.	Mean difference of readings.
5	20	+56.0	5.8
17	5	+33.0	3.0
28	28	-44.0	3.3
50	10	+71.0	7.4
72	10	+43.0	2.5

46 Colour blind.

66 Colour blind.

70 Colour blind.

The first column in the above table gives the number which identifies the observer, the second column the number of readings taken, the third the mean difference between the observer and myself, or Mr. Hadley. The fourth column gives an idea of the consistency of the same observer in reading. It is the mean difference between a single reading and the mean of all the readings, negative differences being counted as positive ones. A glance at the table shows that this mean difference is generally about two or three tenths of a division.

From the results of column 3, Table III has been obtained. Column II gives the number of observers whose readings do not differ by more than half a unit from the numbers given in column I. Column III gives the mean of three successive values of column II, that row against which the number is placed being the middle of the numbers whose mean is taken.

Table III.

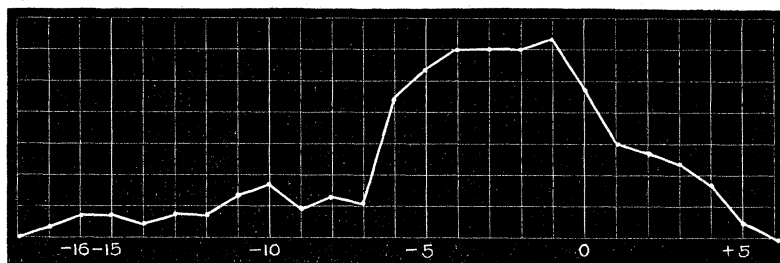
Column I.	II.	III.
-16	1	0.7
-15	1	1.7
-14	3	1.3
-13	0	1.7
-12	2	1.7
-11	3	2.3
-10	2	2.7
-9	3	1.7
-8	0	2.3
-7	4	2.0
-6	2	4.3
-5	7	5.7
-4	7	6.0

Table III—*continued*.

Column	I.	II.	III.
	-3	4	6.0
	-2	7	6.0
	-1	7	6.3
	0	5	4.7
	+1	2	3.0
	+2	2	2.7
	+3	4	2.3
	+4	1	1.7

The results of this table, in which the few observers showing large differences are not included, is plotted in fig. 1, in which the numbers in column I of the above table are taken as ordinates, and the numbers in column III as abscissæ.

FIG. 1.



We see at once that the greatest number of observers read between -1 and -4 . The curve falls rapidly on either side, but there are more observers within the limits of the table apparently showing large negative than large positive differences. Thus, for instance, taking 3 as the mean value of all observers, there was nobody amongst sixty-seven observers differing by eight-tenths or more of a division at the positive, while there were as many as ten differing by the same amount on the negative side. If we were from the given curve to calculate the probability of such large differences, as shown by the five observers, 5, 17, 28, 50, and 72, we should get exceedingly small numbers; this confirms Lord Rayleigh's statement, that differences from normal vision do not seem to follow the law of errors. For differences less than one division of the scale the curve is not unlike the curve of errors, but not for the larger differences; thus half the total number of observers read within 3.5 units of the average. If the difference from normal sight was to follow the law of

errors, we should only have one observer in 50,000 who would read twenty-one units different from the rest. While counting the different members of the same family only as one, I have found three such large differences among seventy-five, and Lord Rayleigh found the same number among thirty observers. He also examined seven female observers, none showing any decided difference from the mean. In the above table numbers 9, 23, 65, and 74 are women, and their readings -4 , -1.5 , -4 , -3 , are very consistent with each other; on the other hand, it must be noted, as a remarkable exception, that 72 is also a woman; her husband has normal sight, but amongst three sons two, viz., 5 and 17, show the same peculiarity.

It is instructive to compare the readings of numbers 28 and 50 with the ratios of intensities given in Table I. It will be seen that, while 28 requires about 2.8 times as much red as green to make yellow, 50 requires nearly five times as much *green* as red to produce yellow. That the ratio of red to green required by one observer is thirteen times as great as that required by the other. How different will compound colours look to these observers! It seems remarkable, however, that both agree in the particular wave-length which they call yellow, and the actual sensation of *pure* colours seems therefore to be the same for both. It seems difficult to explain this fact in any other way than that suggested by Professor Maxwell to account for some peculiarities of his own eyesight, namely, by a selective absorption in the yellow spot of the eye, which differs in different individuals. To judge from the diagram given in this paper, Maxwell had, as compared with his wife, the same peculiarity of eyesight as the different observers mentioned by Lord Rayleigh and number 5 above; this could be explained by greater absorption of green in the yellow spot. But, further, Maxwell's eyes presented an opposite peculiarity for the rays between the green and violet, he wanting *less* green to produce blue than Mrs. Maxwell. This Maxwell tries to explain by more pronounced absorption of the blue rays than of the green. I cannot quite follow him in this explanation, because the greater absorption of blue does not seem to me to affect its position on the colour diagram, but only the intensity of the mixture produced by green and violet. I can only account for this second peculiarity of Maxwell's, that in his case the absorption of the *violet* primary colour was stronger than that of the green. If we adopt the hypothesis that the different position of the pure colours in the colour diagram of different observers is due to an absorption of light in the media of the eye before it reaches the retina, we are at liberty to assume that the sensation of yellow in all eyes is due to an excitation of nerves sensitive to green and red respectively in a fixed proportion.

Direct experiments to determine the absorption in the yellow spot

of the eye have been made by Glan, and it seems to me of importance to repeat and extend his observations. One point, especially, is worth clearing up: in how far are the complementary colours the same for different eyes? As far as I can judge, according to the view just explained, they should be the same.*

I have paid some attention to the possibility of a change in the reading of the same observer at different times; but it is very difficult to obtain decisive evidence in this respect. It is a curious fact, however, that the difference in the reading of the same observer at different times will differ more from each other than one would be led to expect from the consistency of his differing readings taken the same day. As this happens, however, chiefly with observers whose accuracy I have reason to doubt on other grounds, little value can be attached to such differences. It is possible that a careless observer remembers from observation to observation on the same day what he has called a match, though it may be a trifle too green or too red, and in this way the readings may gain an appearance of too great consistency. I take, for instance, number 42, whose readings are characteristic in this respect.

Date.	Number of observations.	Mean reading.	Average difference between each reading and mean reading.
Dec. 4, 1889.....	5	0	2.0
Feb. 13, 1890.....	5	-11	1.9
April 23, 1890....	8	-6	2.6
May 6, 1890.....	8	-10	3.1

On December 4, amongst five readings, he never read lower than -4; while on February 13, he never read *higher* than -4. If I had reason to believe number 42 a careful observer, I should take this as a proof of a change in his eyesight; I am afraid, however, no certain conclusions can be drawn from his observations. But there are some other cases of marked differences.

I have already stated that I have constantly compared my sight with that of Mr. Hadley without being able to trace any decided difference.

The following are a few examples of readings taken at different times with observers in whose judgment I can place reliance:—

* [This is not quite correct, partly owing to the indefinite nature of what we call white. What I meant to say is this: If six pure colours, p, q, r, s, t, u , are related to each other by the equations $ap + bq = cr + ds = et + fu$, then a second observer whose eyes only differed by a different absorption in their media should be able to match the six colours so as to obtain the equations $a'p + b'q = c'r + d's = e't + f'u$. If the resulting colour for the normal eye is white p and q , &c., are complementary colours, but the resulting colour for the second observer would not necessarily appear white to him.—May 23.]

Number 2—

March 4, 1889	—2
Jan. 20, 1890	0
April 23, 1890	—1

Number 4—

March 5, 1889	+5
May 6, 1890	—3

Number 9—

March 6, 1889	—4
March 20, 1889	—3
April 25, 1890	—2
May 7, 1890	—7

Number 14—

March 6, 1889	—7
May 5, 1890	—6

Number 15—

March 6, 1889	+1
May 6, 1890	+3

Number 21—

March 20, 1889	—6
May 2, 1890	—8

Number 25—

March 21, 1889	+4
May 6, 1890	+3

Number 26—

March 24, 1889	—2
May 6, 1890	—7

Most of these observations show no change, and are indeed remarkably consistent. I must except, however, number 4, who is a very trustworthy observer; and number 26 (Professor Dixon). In the latter case, a change seems almost certain either in his eye or in mine. We both observe, however, as Clerk Maxwell has done, that the matches are not quite the same according as one looks straight at the coloured patches, or a little to one side; this would support the view that the absorption in the yellow spot plays an important part.

There is no evidence as to a difference in reading between the right and left eye, except in one case. Generally speaking, the reading

taken with two eyes agrees very well. In the case of one observer, number 30, although no difference could be traced on November 27, 1889, the difference was half a division on May 7, 1890, and sufficient for him to be satisfied that when he made a match with one eye it did not appear a match with the other.

Presents, June 5, 1890.

Transactions.

- Baltimore:—Johns Hopkins University. Circulars. Vol. IX. No. 80. 4to. *Baltimore* 1890. The University.
- Berkeley:—University of California. Library Contents-Index. Vol. I. 8vo. *Berkeley* 1889-90. The University.
- Cracow:—Académie des Sciences. Bulletin International. Avril, 1890. 8vo. *Cracovie* 1890. The Academy.
- Dijon:—Académie des Sciences, Arts et Belles-Lettres. Mémoires. 3me Série. Tome II-III; 4me Série. Tome I. 8vo. *Dijon* 1874-89. The Academy.
- Dublin:—Royal Society of Antiquaries of Ireland. Journal of the Proceedings. Ser. 5. Vol. I. No. 1. 8vo. *Dublin* 1890. The Society.
- Göttingen:—Königl. Gesellschaft der Wissenschaften. Nachrichten. 1889. 8vo. *Göttingen* 1889. The Society.
- Hertford:—Hertfordshire Natural History Society and Field Club. Transactions. Vol. V. Part 8. 8vo. *Hertford* 1890. The Society.
- Leipsic:—Königl. Sächsische Gesellschaft der Wissenschaften. Abhandlungen. Mathemat.-physisch. Classe. Band XV. No. 7-9; Ditto. Philolog.-historisch. Classe. Band XI. No. 6. 4to. *Leipzig* 1889-90; Berichte. Mathemat.-physisch. Classe. Band XLI. No. 2-4; Ditto. Philolog.-historisch. Classe. Band XLI. No. 4; Register zu den Jahrgängen 1846-85 der Berichte und zu den Bänden I-XII der Abhandlungen der Mathemat.-physisch. Classe. 8vo. *Leipzig* 1889-90. The Society.
- London:—British Association for the Advancement of Science. Report of the Fifty-ninth Meeting, Newcastle-upon-Tyne, 1889. 8vo. *London* 1890. The Association.
- London Mathematical Society. Proceedings. Vol. XX. Nos. 368-369. 8vo. *London* 1889. The Society.
- Marine Biological Association. Journal. New Ser. Vol. I. No. 3. 8vo. *London* 1890. The Association.
- Mineralogical Society. The Mineralogical Magazine. Vol. IX. No. 41. 8vo. *London* 1890. The Society.

FIG. 1.

